Stress Testing Best Practices

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Why Stress Tests

In our last note, we discussed how optimization techniques can help fixed-income managers construct portfolios. Moreover, we mentioned that stress testing should be part of an optimization framework that includes hedges and overlays. Whether embedded in a optimization framework or not, stress tests are critical from a risk management perspective. This note, which outlines common stress testing techniques along with some best practices, is our first installment in a series of notes involving stress tests.

Stress tests are designed to estimate the impact of adverse market movements on a portfolio. These market scenarios can be extreme but should always be plausible. For example, changes in monetary policy, increasing inflation, or political instability can be modeled as low-probability events. Meaningful stress tests provide a forward-looking assessment of risk, overcome limitations of simulation models, and help aid the development of risk mitigation techniques.

In fact, since the economic collapse of 2008, regulators have mandated stress tests for large financial institutions to ensure that these institutions have

Types of Stress Tests (non-exhaustive)

- Sensitivity analysis: large shift in *one* factor
- Scenario analysis: simultaneous large shifts in multiple factors
- Historical scenarios (actual events)
- Forward-looking scenarios (plausible, relevant)
- Portfolio-driven (e.g., reverse stress tests)
- Macroeconomic events (e.g., CCAR-type)
- Market events (interest rates, credit spread, FX, etc.)
- Event-driven (event independent of portfolio)
- Worst-case events (terrorist attack, regulatory change, geopolitical)

sufficient capital to continue operations throughout times of economic and financial stress. For instance, under the Comprehensive Capital Analysis and Review (CCAR) framework, large financial institutions in the United States must perform annual stress tests based on the Fed's (forward-looking) macroeconomic scenarios, and disclose the results to the public.

Under a risk management framework, stress tests are an indispensable complement to statistical models such as value at risk (VaR). While the risk factors, distributional assumptions, and pricing functions of VaR models vary, even the most sophisticated approaches are deficient in abnormal or crisis periods. One might be tempted to look further in the tail of the loss distribution for extreme events (with greater percentiles or a metric such as CVaR). But if the loss distribution is derived under normal market conditions, it might not apply in a crisis period. In fact, VaR models do not adequately capture volatility jumps or changing correlation structures and perform poorly when liquidity dries up, as seen by the Lehman crisis.

Stress tests overcome the shortcomings of statistical models. Stress tests need not reflect correlations under normal periods and are designed by specifying directional shocks to parsimonious or granular risk factors. Since stress test results are represented as P&Ls, they are more transparent and intuitive than VaR or CVaR. In addition, they help design better hedges so that managers can mitigate unacceptable levels of risk.

Designing Meaningful Stress Tests

As described above, combining stress tests and statistical models provides a more accurate picture of

Stress Test Design

(1) Select risk factors and model parameters

- Parsimonious risk factors or pricing factors
- Model parameters such as prepayment speeds or calibrated spreads

(2) Apply shifts to risk factors and model parameters

- Instantaneous or path-dependent shifts
- Specify *all* peripheral risk factor shifts or impute from a set of core factors

(3) Reprice using pricing models

- Nonlinear and asymmetric payoffs
- Age securities under path-dependent scenarios

risk. VaR models, along with their risk decompositions, are useful during normal market environments, while stress tests fill the gap during abnormal market periods.

A key component in designing a stress test is the selection of scenarios, which typically requires a subjective projection of risk factors. As a result, practitioners often observe that:

> *"Stress testing is both an art and a science."*

Recall that stress tests estimate the impact of adverse market movements on a portfolio. Forward-looking stress tests should:

- **Be relevant to the portfolio:** Are the correct risk factors selected? For example, applying a -5% shift stress test to *all* equities in a long/short equity portfolio is not useful, whereas this same stress test applied to a portfolio of equity options (with nonlinear pricing) is more useful. Better yet would be a correlated or beta-adjusted shift in equities for both portfolios mentioned above. Other risk analyses involving exposures, sensitivities, and risk factor decompositions aid in identifying concentration and unwanted risk, and should be used in conjunction with stress testing to help select appropriate risk factors for stress scenarios.
- **Examine risk modeling deficiencies:** Is the risk model deficient in any way? If risk factors are not available, then stressing model parameters is helpful. For example, calibrated spreads for illiquid corporate bonds or prepayment factors for mortgage-backed securities can be shocked.
- **Incorporate relevant risk factors:** For example, a

stress test that shifts USD rates should be applied to the full term structure and not to just an individual node, such as the five-year yield. Since the nodes of the yield are correlated, we need to shift all interest rate nodes.

Thus, consistent (and realistic) stress tests involving higher dimensional risk and pricing factors, like volatilty surfaces and the term structure of yield curves, should not violate arbitrage constraints.

- **Include appropriate liquidity horizons:** During a crisis, liquidity is often reduced. A reliable stress test includes liquidity horizons for different asset classes.
- **Combine market and credit stresses:** For example, one can design an anticipated downgrade of an issuer by shifting spreads for bond and CDS holdings that reference the issuer.
- **Use appropriate time horizons:** With a multi-horizon stress test, scenarios are specified over a particular period (and the portfolio is aged accordingly). For example, one might have a view on how the economy may change over the next 12 months. With a fixed-income portfolio, for example, a gradual increase in interest rates over a long horizon will demonstrate significantly different results than an instantaneous shock.

Incorporating Factor-Based Risk Models

Relevant factors need to be selected and shifted under a stress test. Risk analysis typically operates with two sets of conceptual factors:

- **Pricing factors** that are closely related to the market prices of liquid assets and that drive the risk-neutral pricing of those assets in *pricing models*. Such factors include individual equity return time-series or a grid of points from an implied volatility surface.
- **Risk factors** that are more parsimonious, that explain the underlying sources of risk, and that drive the movement of the pricing factors in risk decomposition, simulation, or stress testing.

Risk or portfolio managers who are interested in short-term risk measures on an actively traded portfolio, which may include bets on relative-value trades, typically use risk factors that are very closely aligned with the pricing factors. Such low-level factors may include zero-coupon bond prices derived from yield curves for all of the currencies to which the investor has exposure, price returns for each stock in the portfolio, and points from the implied volatility surface of each option in the portfolio. We refer to this approach as the fine-grained or **granular** approach.

As the investment horizon increases, risk managers typically use less granular, more parsimonious risk factors. The factors that they choose may reflect their institution's investment criteria or their belief that certain fundamental factors drive returns. As an example, a granular approach might use a dense set of key rates to measure the risk of trading along the curve. But for a more long-range view, one might select a parsimonious set of key rates, say the 6m, 1y, 5y, 10y, and 20y nodes.

The selection of risk factors and model parameters should be relevant to a portfolio. Under both approaches, we can group sets of factors along di-

What is a Risk Factor?

In this section, we discussed two types of conceptual factors: pricing factors and parsimonious factors derived from a risk model. We will collectively refer to both as **risk factors**. In fact, we define a risk factor as any time-series of data that is used to compute risk statistics such as VaR.

Sometimes the distinction between pricing and parsimonious risk factors can be blurry. The meaning of a risk factor should be clear from the context.

mensions of risk factor types, such as interest rates, commodities, and credit spreads, and apply shifts along those dimensions.

For correlated stress tests, which we will discuss later in this note, a parsimonious risk model is generally a better choice than a granular model because it helps dampen spurious correlation results.

Nonlinear Positions and Aging

We can combine linear factor models with nonlinear pricing functions. For example, a linear fundamental equity model can be used to define stress scenarios. For example, a style factor such as *growth* can be shocked. The shock is first transmitted to the underlying equity, and then, via nonlinear pricing functions such as Black-Scholes, to all equity options that reference the underlying equity.

After stress scenarios have been selected, positions in a portfolio need to be repriced. Full repricing should be applied to securities that exhibit nonlinear and asymmetric payoffs. A large positive equity return may have little effect on a put, for example, but a large negative return may significantly increase its value. Moreover, if risk managers can quantify the difference between full repricing and delta approximations, they can better analyze tail risks and actively add hedges under "what-if" scenarios. Figure 1 provides an illustration of nonlinear positions that exhibit asymmetric payoffs. Here, we consider a portfolio of stocks, equity options, and equity index options. Relative shifts from -20% to 15% are applied to the underlying stocks and indices, and the results for the portfolio and subportfolios are tabulated. The leftmost plot of Figure 2 displays the P&L values from the equity shifts. This plot also shows the impact on subportfolios such as equity index options. Because of its significant exposure to puts, the overall portfolio exhibits an asymmetric payoff.

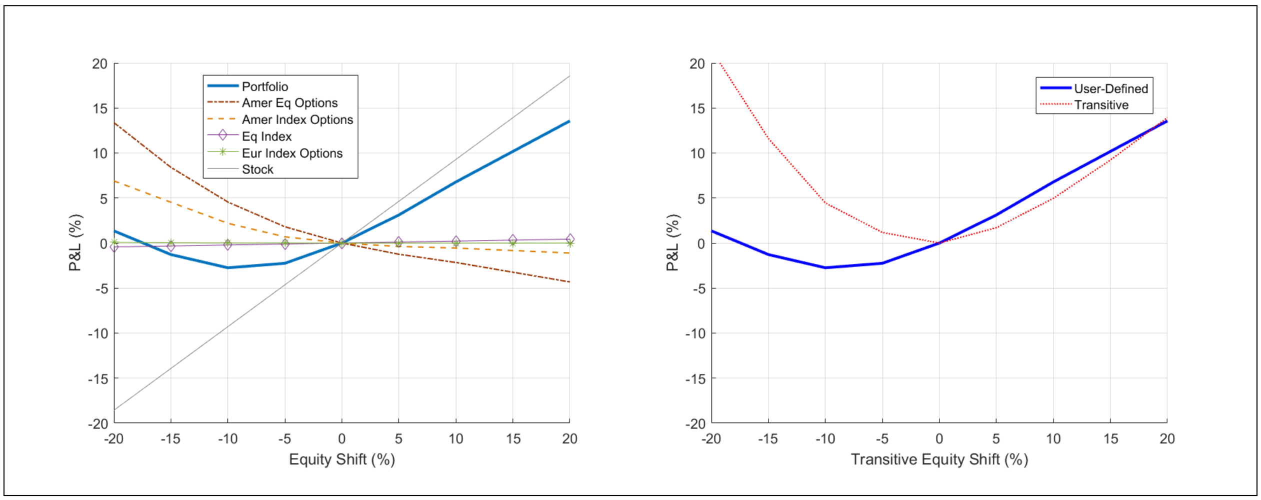
In addition to using the selected risk factor shifts, each stress test applies over a particular time horizon. Instantaneous stress tests, as the name suggests, apply shifts to risk factors without portfolio aging. In this case, the risk profile of the portfolio is unaltered. On the other hand, stress tests applied

Figure 1: Nonlinear Pricing for an Equity Portfolio.

Reporting Levels	PV$	EQ$ -20%	EQ$ -15%	EQ$ -10%	EQ$ -5%	EQ$ 5%	EQ$ 10%	EQ$ 15%
	USD	USD	USD	USD	USD	USD	USD	USD
RS Equity and Options Dec 01 2016	**68,837,633**	**1,292,208**	**-677,929**	**-1,770,770**	**-1,543,551**	**2,242,298**	**4,864,622**	**7,296,933**
American Equity Option (1)	2,911,850	9,408,232	5,908,235	3,188,886	1,278,064	-847,031	-1,408,124	-2,112,186
American Index Option (1)	446,400	4,871,639	3,167,318	1,548,914	434,990	-170,473	-249,124	-373,687
Equity Index (Axioma) (1)	1,490,842	-298,168	-223,626	-149,084	-74,542	74,542	149,084	223,626
European Index Option (1)	75,342	93,145	57,125	31,833	13,597	-10,401	-18,533	-27,800
Stock (1)	63,913,200	-12,782,640	-9,586,980	-6,391,320	-3,195,660	3,195,660	6,391,320	9,586,980

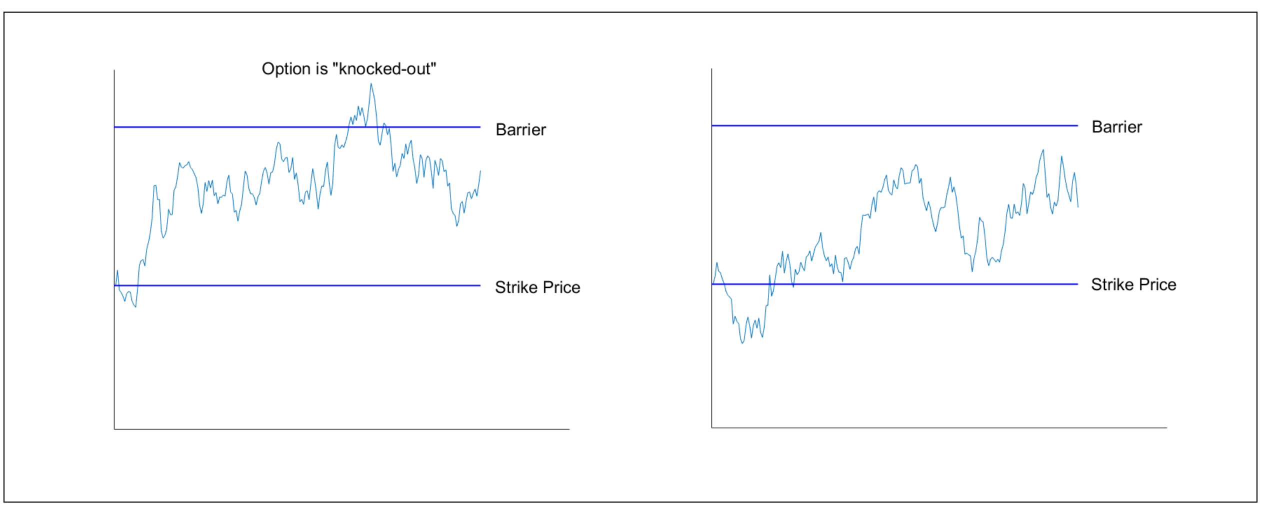
Source: Axioma Risk

Figure 2: Nonlinear P&L Profile for an Equity Portfolio.



Source: Axioma Risk

Figure 3: Path-Dependent Option.



over a time horizon are more realistic since markets do not typically deteriorate over a day but rather over a period such as a week or month. During the time horizon, assets such as options will age. For example, during the course of a one-month risk horizon, an option that expires in two months becomes a one-month option, at which point its risk characteristics and price are quite different from those on the analysis date.

Figure 4 shows results for two historical stress tests for a US fixed-income portfolio that use the same period. In the first stress test, we apply the returns instantaneously, but in the second we apply the returns over the historical duration. That is, we age the positions as we apply the historical returns. In this example, we observe that the loss is slightly lower when we age the portfolio over the entire period (4.25% vs. 4.57%). Positions within the portfolio matured, altering the portfolio's risk profile through time and resulted in a lower loss estimate.[1]

Although we observed little difference between an instantaneous and time-horizon stress tests in Figure 4, large discrepancies will occur for positions that are sensitive to changing volatility regimes. For example, aging equity options when volatility smiles are more pronounced will have a greater impact on P&Ls than an instantaneous shift would otherwise predict. Moreover, large discrepancies can also occur for path-dependent options such as variance swaps and binary options. As an illustration, Figure 3 depicts two scenarios for an underlying index that yield the same cumulative return over the same time horizon. However, each scenario can have different impacts for a path-dependent option, say, for example, an up-and-out barrier option that expires worthless when the barrier is breached.

Standard Techniques

Stress tests can be historical, user-defined, or transitive on a set of (stressed) core risk factors. In addition to shifting risk factors, model parameters can be shifted.[2] Model parameter shifts are useful for event-driven and worst-case event shocks. For example, the impact of different cumulative default rates (CDRs) can be applied to a structured debt portfolio, a jump-to-default statistic can be applied to a risky issuer, or credit spread shocks for illiquid securities can be analyzed.

Historical Stress Tests

Historical stress tests replay past market movements to a current portfolio. Of course, these stress tests are useful only if one believes that past events may repeat.

Constructing a historical stress test is straightforward. First, one selects the historical start and end dates. Next, one applies the historical returns of risk factors to reprice the portfolio. Typically, we apply the returns to the current base price of securities in the portfolio.

Subtly, the choice of risk models (and consequently, risk factors) affects historical stress tests.

[1] The matured bonds were trading slightly below par at the analysis date and were reinvested in cash.

[2] In fact, both risk factor and model parameter shifts should be part of user-defined stress tests. Model stress tests allow one to keep arbitrage constraints valid (e.g., volatility surfaces).

For fixed-income positions, we typically input the movements of risk factors, such as interest rates or issuer spreads, into a pricing function, instead of examining *actual* bond returns. Thus, historical stress tests based on fixed-income risk factors can be applied to recently issued bonds that did not exist in the historical period. For equities, however, it is more common to apply *actual* historical equity returns to a current equity position. In this case, we are utilizing the granular model. Alternatively, (as with fixed-income positions), we could use a factor-based approach to stress equities. Under a fundamental equity factor model, for example, we could apply the historical returns of factors (including specific returns) to the *current* exposures. The level of discrepancy between a fundamental and granular equity model depends on how much the exposures vary over the historical period. Here, we are asking what would happen to a portfolio by applying historical factor returns to today's exposures. For example, for a firm with recently increased leverage, historical leverage returns have a greater effect than what a granular approach would suggest.

Moreover, a stress test based on a risk factor model also provides an attribution to factor returns. This is a useful way to understand and decompose historical returns into systematic components.

User-Defined Stress Tests

User-defined stress tests are simple in concept: They apply a specified shift of risk or pricing factors to a portfolio. User-defined stress tests do not say anything about correlations, rather, they simply shift risk factors and reprice the portfolio. One advantage of user-defined stress tests is that the user has control of which risk factors to move. For example, the individual key rates of a yield curve can be shifted to analyze flattening or steepening of interest rates.

Although user-defined stress tests can give (virtually) complete control of shifting risk factors, addressing the movement of all risk factors in a portfolio is unwieldy. Instead, grouping risk factors and applying shifts to these groups is much more manageable. For example, we can examine the shift in categories such as interest rates, spreads, FX rates, and implied volatilities, in isolation or in combination. Moreover, user-defined attributes, such as internal ratings or trading strategy descriptions, help risk and portfolio managers apply targeted stress tests that are more consistent with the investment strategy of the portfolio.

As an illustration of user-defined stress tests, consider Table 1. The portfolio is comprised of (i) stocks from France, Canada, and the US; (ii) equity options on the SPY; and (iii) US Treasury bonds. The risk report in Table 1 is grouped by the country of risk. The second and third columns are stress tests involving a shift in equity prices and interest rate yields by -5% and 100 bps, respectively. Computing shocks for the stock portion of the portfolio is trivial (and does not require a risk system). For instance, we see that a -10% equity shift simply translates to a loss of 2.2% for the initial holding of French stocks (22%). For nonlinear positions in this report (and also for the nonlinear positions in Figures 1–2), we applied full repricing using the Black-Scholes formula. Likewise, for the fixed-income portfolio we applied full repricing for the parallel shift (100 bps) of the interest rate

Figure 4: Historical Stress Test.

Reporting Levels	Present Value	Sept-Oct 2008 (Inst 09/08-10/08)	Sept-Oct 2008 (Path - 09/08-10/08)
	USD	USD	USD
US Fixed Income PF Blend Jan 29 2016	**100,017,271**	**-4.57**	**-4.25**
A_A_A (3)	17,114,235	-0.39	-0.38
Bond (4)	4,713,828	-0.24	-0.24
JOHNSON & JOHNSON - 3.375% Sr Nts Due 2023 (1)	99,093	0.00	0.00
MICROSOFT CORP - 3.00% Sr Nts Due 2020 (1)	334,905	-0.01	-0.01
MICROSOFT CORP - 4.200% Nts Due 2019 (1)	188,921	0.00	0.00
UNITED STATES TREASURY - 30-Yr Bds Due 05/15/2042 (1)	4,090,909	-0.23	-0.23
Callable Bond (7)	2,400,318	-0.07	-0.08
MBS Pool (141)	10,000,089	-0.07	-0.07
A_A (3)	17,878,108	-0.53	-0.53
A (2)	17,695,908	-0.66	-0.66
B_B_B (2)	2,329,021	-0.07	-0.07
B_B (3)	7,317,410	-0.47	-0.43
B (2)	22,109,655	-1.34	-1.21
C_C_C (2)	15,218,829	-1.08	-0.94
C_C (1)	340,983	-0.03	-0.02
C (1)	13,123	0.00	0.00

curve.

Transitive Stress Tests

More sophisticated than user-defined stress tests are transitive stress tests. Transitive or conditional stress tests are correlated stress tests.

A set of core factors is explicitly selected and shifted while the movement of the remaining peripheral risk factors is inferred from the covariance of factor returns. (See, for example, Kupiec [3].) Core factors need not be part of a risk model, but should be relevant to the portfolio from an asset class or macroeconomic perspective. For example, we might specify a 12% movement in oil prices in a transitive stress test. Not only will the price of oil move in this stress test, but all other risk factors such as interest rates, equity prices, and commodity prices will also move. The direction and magnitude of all peripheral risk factors is determined from their volatilities, and correlations to the core factor, in this case the return in oil prices. Clearly, transitive stress tests are statements about correlations and depend on the look-back period that is used.

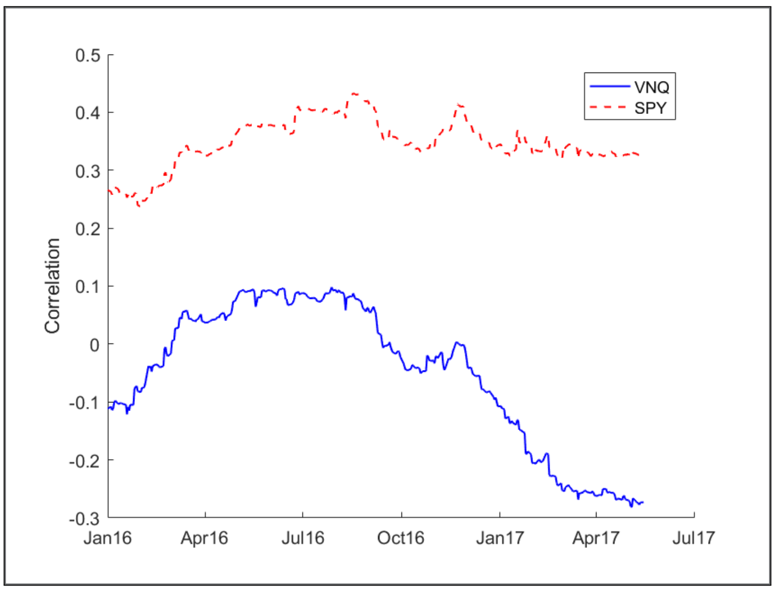
Table 5 provides an example of a sensitivity analysis using a transitive stress test. We examine the interest rate sensitivity on the analysis date 15-May-2017 for two portfolios: SPY (S&P 500 ETF) and VNQ (Vanguard REIT ETF). We shift the five-year interest rate yield, which is the core factor, by 200 bps and tabulate the P&Ls. As expected, we observe a positive P&L for SPY since the equity and interest rate returns are positively correlated in this period. On the other hand, VNQ exhibits a negative return since its returns are negatively correlated to interest rate returns. See Figure 6 for a plot of rolling correlations to the five-year yield return. Note that different look-back periods can generate P&L results with opposite directions. We see this from Figure 6, where the correlations between VNQ and interest rate returns were both positive and negative.

It is useful to decompose transitive stress tests into two components. The first component is the selection and movement of core factors. We can even specify a type of historical stress test based on the

Figure 5: Interest Rate Sensitivity for REIT Portfolio on 15-May-2017.

Reporting Levels	PV$	IR+200bps
	USD	USD
RS TEST May 15 2017	**2,000,000**	**-357.56**
Equity Index (Axioma) (2)	2,000,000	-357.56
SPY (1)	1,000,000	1,659.25
VNQ (1)	1,000,000	-2,016.81

Figure 6: Rolling One-Year Correlation to Interest Rates.



actual historical movements of core factors. The second component is the *market context* component where volatilities and correlations of peripheral factors are specified. Here, one can use different periods to estimate volatilities and correlations. For example, to compute correlations, we can use the current period from the analysis date; a crisis period where correlations between risk factors were greater; or, under a historical core factor stress test, the same historical period including the core factor shifts.

Transitive stress tests allow a parsimonious set of factors to be chosen and have the following advantages:

- They enable risk and portfolio managers to construct forward-looking scenarios. They allow managers to test portfolios under scenarios that may have never occurred but that reflect the managers' views.
- They enable managers to examine relationships among different asset classes such as equities, fixed-income, FX, and commodities.
- They enable historical stress testing (using core factors) of assets that did not exist in the selected historical period.

Although this type of stress test is extremely useful, the user should beware of spurious results. When just one core factor is selected, as in Figure 5, computing betas for all peripheral factors is useful. Other examples of transitive stress tests with one core factor appear in Table 1. Transitive stress tests depend on correlations; we use a one-year look-back period to compute correlations in this report. Here, we shock the S&P 500 index by -10% and apply its shift to all risk factors for the portfolio. This shift no longer translates to a -10% for stocks, but rather a correlated shift is applied. For instance, we have a loss of 2.8% (instead of a loss of 2.2%) for French stocks due to a beta greater than one.

The rightmost plot of Figure 2 provides another example of a transitive stress test involving one core factor. Here, we apply transitive shifts of the S&P 500 index, which is represented as a dashed line. Note that the downside put protection significantly increases under the transitive stress test. The underlying stocks corresponding to the puts have a large beta to the S&P 500 index, with an average beta of 1.8. Thus, these underlying betas amplify the downside shift in the core factor and result in a larger gain in put values.

These examples of transitive stress tests help illustrate a couple of points from a practitioner's perspective. First, comparing user-defined and transitive stress tests side-by-side is useful, and second, intermediate statistics, such as correlations and betas, should accompany and help explain differences between transitive and user-defined stress tests.

Betas and correlations should also be examined when shifting multiple core factors. A diagnostic report should also be included to detect whether core factors are highly correlated or not. Shifting highly correlated core factors in opposite directions can produce spurious results, and in this situation, the number of core factors needs to be reduced.

Partitioned Transitive Stress Tests—Preserving and Breaking Correlations

Partitioned transitive stress tests are composite transitive stress tests that are applied to a portfolio partition. For example, we can partition a portfolio into different asset classes. For each partition, we apply separate *localized* transitive stress tests, and then sum the results to the portfolio level.

This allows us to *preserve* correlations within partitions, while shifting core factors across different partitions in arbitrary ways, which may even *break* correlations between core factors. Loosely, partitioned stress test allow us to preserve and break correlations in a meaningful way. See Figure 7 for a depiction. For example, we can partition a multi-asset portfolio by asset classes such as fixed-income, equities, and commodity futures. For each partition, we select core factors, such as the five-year government yield for the fixed-income partition; the S&P 500 index for the equity partition; and the GSCI for the commodity partition. When we shift the fixed-income core factor, all other yields will move in a correlated fashion, impacting all fixed-income pricing, and likewise with the other two factors. Although equity and yield returns are typically positively correlated, we might want to shift these factors in opposite directions, which we otherwise would not be able to achieve under a global transitive stress test. This might be a useful stress test for an anticipated increase in inflation, during which asset prices would typically move together.

One side benefit of partitioned transitive stress tests compared with transitive stress tests is that they reduce spurious results. By applying local stress tests, we typically use fewer core factors at *once* than under a global transitive stress test, and thus we reduce the possibility of selecting correlated factors.

We provide examples of partitioned stress tests in columns L1–L3 of Table 1. Correlations between core and peripheral factor returns were computed using a one-year look-back period from the analysis date. The portfolio is partitioned by country of risk (FR, CA, US). The US partition is further partitioned into equities and fixed-income. The core factors are: TSX index for CA; EWQ (iShares MSCI France ETF) for FR; S&P 500 for US-Stocks/Stock Options; and USD 5y yield for US-Treasury partitions. The local transitive shift definitions are provided at the bottom of Table 1. L1 represents a mild global downturn where all assets decrease in value; L2 represents a weakening of French stocks and strengthening of North American stocks; and L3 represents a potential scenario for the 2017 election in France.

The market dislocation L2 scenario was based on a historical period in 2014 when French and North American stocks briefly moved in the opposite direction while correlations remained positive—a result that we could not capture under a global transitive stress test.[3] Note that we have a total of four core factors in this example, but under each partition, only one core factor shift is applied.

[3] The core factor equity shifts are based on the period from June to July 2014. Even though the S&P500 and EWQ indices moved in opposite directions, the rolling one-year correlation remained positive over this period.

Figure 7: Partitioned Transitive Stress Test.

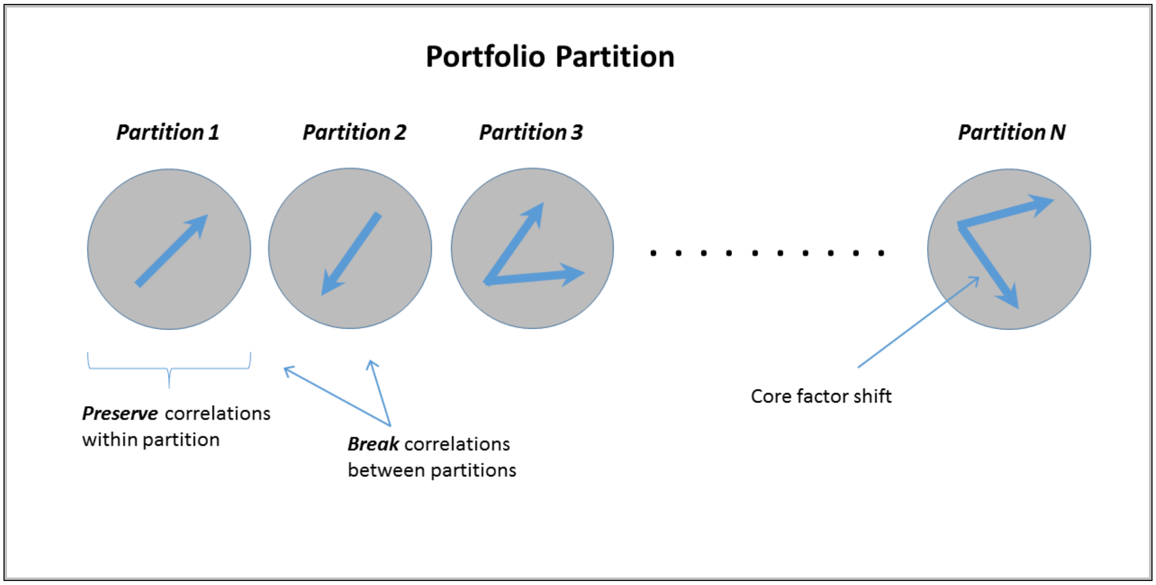


Table 1: Stress Tests for Hypothetical Portfolio on 13-Mar-2017.

		User-Defined		Transitive			Partitioned Transitive		
Portfolio	PV (%)	EQ (-10%)	IR (100bps)	SPX (-10%)	TSX (-10%)	IR (100bps)	L1	L2	L3
Total	100.0	−12.2	−1.4	−12.3	−7.5	9.9	−12.9	−1.2	−5.5
FR									
Stocks	22.2	−2.2	0.0	−2.8	−1.9	0.1	−3.6	−2.5	−5.0
CA									
Stocks	16.5	−1.7	0.0	−1.5	−1.7	0.1	−1.3	0.8	0.2
US									
Stocks	22.2	−2.2	0.0	−2.3	−0.9	3.2	−1.6	0.3	0.0
Stock Options	7.7	−6.1	0.0	−6.1	−3.1	7.8	−5.0	1.5	0.0
US Treasury	31.3	0.0	−1.4	0.3	0.0	−1.3	−1.4	−1.4	−0.7

Transitive (Local)	Description	FR	TSX	SPX	IR
L1	"Mild Downturn"	-13%	-8%	-7%	+100bps
L2	"Market Dislocation"	-9%	+5%	+1.5%	+100bps
L3	"French Election 2017"	-18%	+1%	0%	+50bps

Table 2: Comparison of Standard Techniques.

Stress Test Type	Pros	Cons
Historical	Easy to implement. No *explicit* distributional assumptions. Applies to both granular and factor-based approaches.	The history may not apply.
User-defined	No correlation assumptions required. Any risk factor can be shifted. Grouping can be applied.	Can be unwieldy. Arbitrage constraints can be violated.
Transitive	Correlated stress tests are more realistic. Can embed historical stress tests via core factor shifts.	Spurious results can occur.
Partitioned/Local Transitive	More control/flexibility than transitive over correlation structure. Correlation can be broken in a meaningful way.	Spurious correlations can occur within partitions.

Best Practices

In the previous sections, we outlined standard stress tests and provided some examples. So which ones should we use?

Table 2 summarizes the advantages and disadvantages of each type of stress test. For instance, historical stress tests are easy to implement but are they relevant to the current portfolio? If we believe that they are irrelevant, then we might argue that they are completely useless and are imposed by only regulatory or investor pressure. Yet even if a historical event does not repeat *exactly*, historical periods still aid in extracting correlations for transitive stress tests and help design forward-looking stress tests. In addition, historical periods can give us a sense of how plausible risk factor shocks are.

Partitioned transitive stress tests should align with our intuition. In one sense, there are fewer *moving parts* because we are applying correlated shocks locally, using fewer core factors. For example, we could have taken the last examples L1–L3 from Table 1 and instead applied *one* transitive test using all four core factors simultaneously. When multiple factors are used, we might obtain noninuitive results from a multilinear regression. Mathematically, the results may be correct, but this is not helpful from a practitioner's perspective. Well-designed partitioned transitive stress tests, like user-defined stress tests, should provide P&L results that move in the expected direction.

So what are some best practices for historical, user-defined, and partitioned transitive stress tests? Expanding on the pros and cons listed in Table 2, we propose the following guidelines:

- **Use history:** Historical stress tests are useful if one believes that history will repeat. However, as mentioned above, historical periods can also be used to design other stress tests. The example L2 from Table 1 was based on a historical period when French and US stocks moved in opposite directions. Interestingly, the correlations using a one-year period were still positive.

- **Use transitive and user-defined stress tests together:** User-defined stress tests, no matter how trivial or simple, should be used alongside all transitive stress tests. No correlations are involved for user-defined stress tests, which require only a useful partition of the portfolio. The difference between user-defined and transitive stress tests is due to correlations and volatilities of peripheral risk factors. Examples with user-defined and transitive stress tests displayed side by side are provided in Figure 2 and Table 1.

  In both user-defined and local transitive stress test, P&L results should directionally align with our intuition involving exposures and risk contributions.

- **Update stress tests on a regular basis:** Stress tests need to reflect changing market conditions and portfolio composition, and as such need to be updated during these periods.[4]

  Recall that stress tests are more intuitive than VaR since they are represented as P&Ls. Thus, they are easier to understand and additive across multiple portfolios.

- **Use multi-horizon with instantaneous stress tests:** Typically markets do not deteriorate instanta-

[4] In contrast, VaR models are not updated as frequently as stress tests.

neously but rather over a time horizon such as a week or month. During this period, assets will age and the risk profile of the portfolio may significantly change. For instance, over a month horizon, an option with two months to maturity becomes an option with one month to maturity. During periods of pronounced implied volatility, one can observe significant differences between instantaneous and multi-horizon stress tests. Also, path-dependent options are sensitive to multi-period scenarios.

Note that, in addition to applying the same incremental changes over a given period, one can incorporate front- and back-loaded stress scenarios.

- **Diagnostics:** For transitive stress tests, both global and local, one should examine peripheral factor betas. A diagnostic report detecting highly correlated core factors is also important. When correlated core factors are present, the number of core factors needs to be reduced.

  Diagnostics of marginals (both core and peripheral risk factors) allows us to assess the plausibility of the size of the shocks. Again, historical data will provide guidance on whether these shocks are plausible or not.

  Fitting different copulas to the marginal distribution of risk factors can provide some guidance on the likelihood that a stress test scenario will occur, a topic for a follow-up note on stress testing.

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