Axioma In-Practice Series

# Fixed-Income Portfolio Optimization

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## Introduction

Risk models are essential for risk management. They quantify and help analyze the embedded risks of portfolios by identifying systematic and firm-specific components of risk. In particular, exposure and risk contribution analyses identify different types of risk such as interest rate or equity risk, and helps portfolio managers mitigate risk by adding a hedge or selling concentrated positions. As a decision support tool, risk models aid in portfolio construction, performance attribution, and scenario analysis.

By using a risk model to analyze a portfolio, managers gain insight into risk and exposures. For example, how would an increase in spread duration in the energy sector impact the risk of a portfolio? Or what is the impact of an overweight to Financials relative to a benchmark? Fixed-income risk models allow us to quantify these questions, which in turn help managers make better decisions around on how they construct and hedge their portfolios.

### Why Optimization?

Having knowledge of your risk exposures is not

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always enough—trading eventually must be done to enact views into the portfolio while trading off multiple goals. Managers must decide which bonds will best implement their views while not causing other unwanted exposures, make sure the trade list is in tradeable increments, and not incur too much market impact and/or transaction costs. Combining optimization techniques with a fixed-income risk model aids in portfolio construction to help achieve the multifaceted objectives of a fixed-income manager. Axioma's optimizer goes beyond traditional Markowitz mean-variance optimization (MVO) and allows fixed-income managers to understand their portfolio risks and build portfolios more in line with their desired risk profile in an efficient manner. We will highlight this type of workflow by using Axioma's fixed-income factor model and optimizer on a sample fixed-income portfolio where we replicate an index with constraints and perform an immunization. In summary, fixed-income optimization allows us to:

- Tilt towards desired exposures
- Control (unwanted) exposures
- Rebalance/construct portfolios
- Quantify deviation from a benchmark
- Implement manager strategies
- Add hedging and overlay strategies
- Control for liquidity and transaction costs

However, fixed-income portfolio construction and rebalancing is challenging since most fixed-income indices contain a significant number of illiquid securities. Nonetheless, we will take a pragmatic approach and remain cognizant of modeling limitations.

## Fixed-Income Investment Strategies

Fixed-income strategies are broadly classified as active or passive. Active strategies attempt to beat the market by trying to anticipate interest rate movements, firm-specific events, or to identify relative mispricing. On the other hand, passive strategies do not attempt to beat the market, but are designed to match the composition and performance of an index, such as the Barclays US Aggregate Bond Index. Passive strategies that track an index are also referred to as *indexing* strategies. Typically, tracking error is constrained within a small risk budget to ensure that the portfolio closely tracks the index.

### Optimization challenges

Fixed-income indexing strategies remain popular for investors, especially with the trend toward lower management fees. However, portfolio construction and rebalancing for these strategies is challenging. Index replication is difficult since most fixed-income indices contain thousands of securities, of which a significant portion are illiquid and have difficult to estimate fair market prices. In addition, bonds maturing, new bond issuances, and reinvestment of coupon payments all facilitate frequent rebalancing, which can be expensive and more tedious than equity index replication.

Thus replicating a bond index *precisely* is not possible. Instead, a **stratified sampling** approach is used where a manager divides an index into "cells" that represent different characteristics of the index. For example, bonds can be grouped by maturity buckets, sectors, coupon rates, and credit risk; and a portion of bonds from each group or cell is used for portfolio construction. Further, constraints involving a manager's preferences and institutional mandates, such as liquidity constraints and credit quality, need to be enforced in the risk model.

Although we will employ an optimization approach, one should be aware of its limitations. The optimization framework that is widely used in equity portfolio management may not be suitable for certain fixed-income portfolios; in particular, portfolios with nonlinear positions that exhibit asymmetric returns. Because linear factor models employed in optimization are based on Gaussian returns, they will not capture asymmetric returns, and thus are not suitable for *highly* nonlinear portfolios. As long as the portfolio can be approximated well under a parametric approach,[1] an optimization approach will be appropriate. Nonetheless, from a risk management perspective, one should complement any optimization approach with tools such as stress testing and scenario analysis.

Fixed-income securities also have larger minimum holding sizes and minimum trading sizes, which can be difficult for some optimizers to handle. Thus optimizers need to handle these hard-to-solve constraints, despite that they make rebalancing more difficult than it would be in an ideal world where partial and small shares could be traded.

Although replication of an index or fund is a useful application of optimization, it should be acknowledged that we can use optimization for hedging (see, for example, [2]) and construction of custom indices. For example, creating custom fixed-income portfolios is useful for sell-side firms, where they receive order flows from clients and construct a basket of bonds based on client preferences such as sectors or credit quality. Other examples include the hedging of FX risk from bonds that pay coupons in foreign currencies using FX forwards, hedging credit risk using CDS contracts, or interest rate risk using interest rate swaps.

## Sample Portfolios

The portfolios we consider are the iShares iBoxx High Yield Corporate Bond ETF (HYG) and an emerging market sovereign portfolio based on the JPM EMBI. We will provide point-in-time and time series (back test) analysis of these sample portfolios throughout this note.

Figure 1 provides a snapshot of risk for HYG on the analysis date 10-Jan-2016. Exposure and sensitivities (via duration) are provided. By market exposure, we observe that the first four sectors (Communications, Energy, Consumer Non-Cyclical, Consumer Cyclical) in the report contribute to more than 60% of the portfolio by market value. As such, we expect the majority of risk to come from these sectors. The last two columns display risk contributions for VaR at the 95% confidence level for full repricing (mVaR95) and a linear model (mVaR95 Linear).[2] These results are annualized and displayed as a percentage. From a risk

[1] One can compare risk methodologies based on parametric and full-pricing (Monte Carlo) to gauge the appropriateness of using an optimization framework.

[2] Note that linear VaR is greater than Monte Carlo VaR. This is in part due to the nonzero recovery rate settings that are embedded in the Monte Carlo simulations.

Figure 1: Risk Profile for HYG 10-Jan-2017 generated from Axioma Risk

| Reporting Levels | Weight (%) | OAS (bps) | VG01 ($) | Duration | Spread Duration | mVaR95 | mVaR95, Linear |
|---|---|---|---|---|---|---|---|
| | USD | USD | USD | USD | USD | USD | USD |
| **RS HYG Jan 10 2017** | **100.00** | **349** | **-41,413** | **3.76** | **3.78** | **4.64** | **5.59** |
| Communications (197) | 24.81 | 371 | -12,928 | 3.74 | 3.74 | 1.01 | 1.49 |
| Energy (184) | 14.46 | 356 | -4,571 | 4.25 | 4.27 | 0.99 | 1.06 |
| Consumer Non-Cyclical (124) | 13.32 | 425 | -4,607 | 4.03 | 4.04 | 0.46 | 0.63 |
| Consumer Cyclical (137) | 12.40 | 286 | -4,711 | 3.67 | 3.68 | 0.37 | 0.53 |
| Technology (75) | 7.62 | 317 | -5,018 | 3.31 | 3.32 | 0.22 | 0.26 |
| Capital Goods (82) | 7.38 | 259 | -3,682 | 3.64 | 3.64 | 0.34 | 0.44 |
| Basic Industry (73) | 5.87 | 344 | -1,739 | 3.51 | 3.51 | 0.53 | 0.55 |
| Electric (40) | 3.35 | 446 | -1,991 | 4.09 | 4.09 | 0.41 | 0.17 |
| Finance Companies (33) | 3.27 | 256 | -272 | 3.13 | 3.13 | 0.09 | 0.14 |
| Banking (25) | 2.56 | 246 | -11 | 4.09 | 4.27 | 0.10 | 0.14 |
| Transportation (16) | 1.13 | 402 | -423 | 2.96 | 2.97 | 0.02 | 0.04 |
| Insurance (12) | 1.05 | 374 | -519 | 3.53 | 3.68 | 0.03 | 0.05 |
| Industrial Other (8) | 0.75 | 747 | -85 | 2.78 | 2.79 | 0.01 | 0.03 |
| Financial Other (7) | 0.71 | 313 | -261 | 1.91 | 1.91 | 0.01 | 0.02 |
| Reits (8) | 0.50 | 253 | -477 | 4.81 | 4.81 | 0.02 | 0.02 |
| Cash (1) | 0.31 | 0 | 0 | | | 0.00 | 0.00 |
| Owned No Guarantee (3) | 0.28 | 507 | -60 | 3.58 | 3.58 | 0.02 | 0.03 |
| Brokerage/Asset Managers/Exchanges (4) | 0.22 | 437 | -58 | 3.97 | 3.98 | 0.01 | 0.01 |

Figure 2: Risk Factor Decomposition for HYG 10-Jan-2017 generated from Axioma Risk

| Reporting Levels | VaR95, Linear | mVaR95, Linear |
|---|---|---|
| | USD | USD |
| **RS HYG Bmk Jan 10 2017** | **5.59** | **5.59** |
| Risk Type : Interest Rate (2) | 3.48 | -0.25 |
| Risk Type : Issuer Credit (29) | 6.38 | 5.69 |
| Risk Type : Vega (1) | 0.64 | 0.23 |

contribution perspective, we see (as expected) that the top four sectors by exposure contribute more than 65% of total risk.

Figure 2 provides a risk factor drilldown for the HYG portfolio. The column labeled "VaR95, Linear" displays standalone results where we isolate the movement of a particular risk type. For example, the row "Risk Type: Interest Rate" only examines the risk of interest rates in isolation; in this case it represents 3.45% of the risk on a standalone basis. The sum of the standalone risk numbers is greater than the total, indicating a diversification benefit across risk factors. The last column, displays risk contributions, which by definition sums to the total portfolio risk. Diversification is present from negative correlations between interest rate and spread factors.

## Factor Model

The reports in Figures 1 and 2 are driven from a fixed-income risk model. The fixed-income model that we utilize incorporates systematic factors such as interest rates and a hierarchy of spreads such as swap spreads, rating/sector spreads, and issuer spreads; see Figure 3. In addition pricing factors such as implied volatilities are included. (This is represented as vega risk in risk factor drilldown report in Figure 2.)

The fixed-income factor can be represented succinctly as

$$r = Xf + \varepsilon \tag{1}$$

where $r$ is the vector of bond returns, $X$ is the exposure matrix, $f$ are factor returns, and $\varepsilon$ are specific returns.[3] The systematic factors are comprised of interest rate key rates (1y, 2y, 5y, 10y, 20y) and sector/rating spreads based on GICs classifications, such as Information Technology/Sub-Investment Grade. Exposures are price sensitivities; the interest rate exposures are key rate durations, and spread exposures are spread durations. The idiosyncratic components are estimated from issuer-specific spreads.

The spread hierarchy represented in Figure 3 provides a risk decomposition that is intuitive. Systematic factors appear first in the hierarchy, followed by more dependent factors that are related to the issuer. As an illustration, consider Figure 4 where the spread volatility (34bps) of a corporate bond is decomposed by its spread factors.[4]

## Portfolio Construction

Axioma's optimizer is flexible enough to handle a wide range of fixed-income portfolio construction cases. In this paper we will review three main variations: (i) Portfolio Replication (i.e. passive), (ii) Factor Tilts (i.e. active), and (iii) Hedging and Immunization.

Liquidity is a key component that must be addressed under fixed-income portfolio construction and rebalancing. Liquidity itself can have multiple interpretations, from a broad economic perspective involving the notion of flows to a market perspective involving the ability to trade assets effectively. We will consider the latter, where trading liquidity relates to the risk of transacting in a bond.

[3] Equation (1) has been adjusted for carry and roll.

[4] In Figure 4, the specific risk of 14bps is estimated from the excess issuer spread.

Figure 3: Fixed-Income Spread Hierarchy

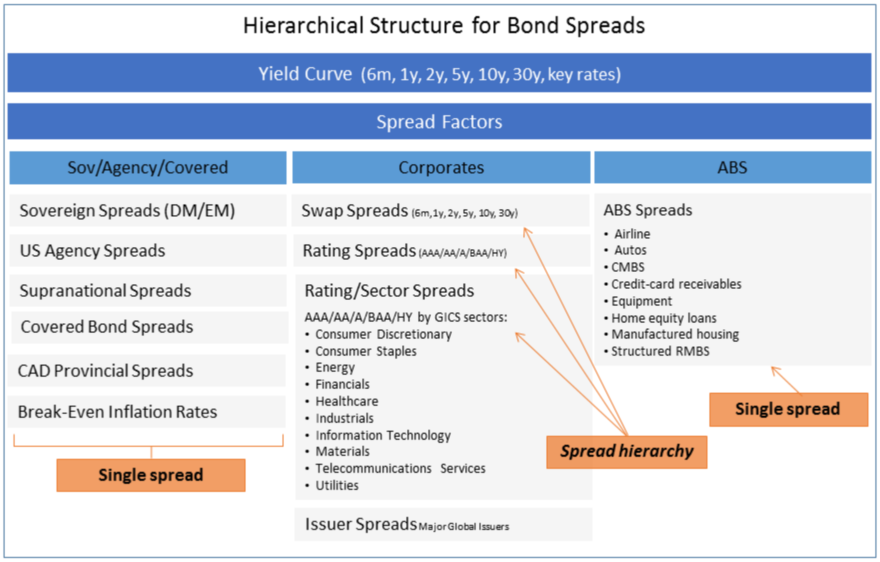
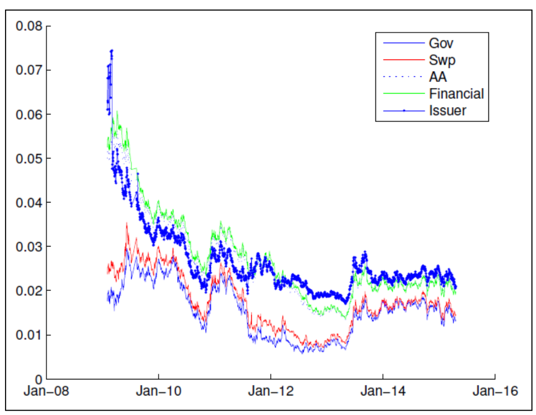



Figure 4: Fixed-Income Spread Hierarchy Plot




| | Risk Contribution (bps) |
|---|---|
| **Total Spread VaR 95%** | 34.91 |
| **Swap Spread** | 18.29 |
| **Rating A** | 2.13 |
| **Sector Financials** | 0.43 |
| **Excess Issuer Specific** | 14.06 |

Unlike for equities, liquidity measures for bonds based on trading volume alone are inappropriate because a traded bond is not necessary a liquid bond, for example, forced selling of fallen angel bonds. Along with traded volume, liquidity can be estimated from bid-ask spreads (which measure round trip transactions), market depth, and frequency of transactions. Market depth represents the number of shares that can be traded at a given price without adding extra costs above the bid-ask spread.

Constructing fixed-income liquidity scores is thus difficult, but nonetheless, constraints based on liquidity scores should be the starting point for portfolio construction and rebalancing.

As an illustration, we construct a portfolio to replicate the HYG index. We utilize liquidity indicators from IDC (Interactive Data Corporation). Trading liquidity is defined as the ability to exit a position at or near the current value. IDC's methodology produces a forward-looking estimates for traded volume capacity (a measure of depth), which in turn are used to produce liquidity scores and projected days to liquidate.[5] Figure 5 represents a sample liquidity report. The column "IDC Liquidity Score" is a relative liquidity score based on sectors, where scores range from one to 10, with one being the most liquid.

Our universe is comprised of the constituents of the HYG index. However, we constrain our portfolio to a tradeable universe, which is a subset of the universe. We allow the optimizer to buy more liquid securities based on IDC liquidity indicators along with the following constraints:

- Restrict IDC liquidity score to 1-8
- Restrict position holding exposure to no more than 5% of the portfolio value
- Match duration of index

This example represents replication under a passive strategy based on liquidity scores. In fact, the liquidity constraint above restricts the trade list to approximately 60% of the securities within the benchmark. This constraint might represent an institutional mandate that enforces sufficient liquidity during a crisis period. Figure 6 lists these constraints from Axioma's optimizer. The results of the optimization are provided in Figure 7. The initial portfolio is the HYG portfolio with a total standard deviation of 3.94%. The optimized portfolio has a total standard deviation of 3.98% with an active standard deviation of 22bps relative to HYG. The effective duration increased from 3.75 to 4.0; thus an increase in total risk is expected.

In addition to the example above involving replication under a passive strategy based on liquidity scores, we can consider other replication strategies such as a pure linear approach minimizing active exposures and modified stratified sampling constraints using an optimizer. Under a modified stratified approach we can purchase bonds from certain critical buckets or attributes such as exposure and coupon payments.

## Factor-Tilted Portfolios

At times, portfolio managers want to deviate from a benchmark and impose their *tilts* on their portfolio. For example, if a portfolio manager believes

[5] IDC's approach to measuring liquidity involves the use of statistical techniques to estimate future potential trading volume and price uncertainty.

Figure 5: Sample IDC Liquidity Report for HYG 10-Jan-2017

| Reporting Levels | PV (%) | IDC Liquidity Ratio | IDC Liquidity Sector | IDC Trading Volume Cap | Days to Liquidate |
|---|---|---|---|---|---|
| | USD | | | | usd |
| **RS HYG Jan 10 2017** | **100.00** | | | | |
| Cable Networks (11) | 1.40 | | | | |
| CSC HOLDINGS LLC - 10.125% 144A Sr Nts Due 2023 | 0.24 | 0.000387922 | 10 | 8155806.29 | 6 |
| CSC HOLDINGS LLC - 10.875% Due 2025 | 0.26 | 0.000455407 | 10 | 6832980 | 7 |
| CSC HOLDINGS LLC - 5.25% Ser B Sr Nts Due 2024 | 0.09 | 0.003110826 | 5 | 1537899.196 | 11 |
| CSC HOLDINGS LLC - 5.500% 144A Sr Guar Nts due 2027 | 0.15 | 0.001022304 | 8 | 3997929.272 | 7 |
| CSC HOLDINGS LLC - 6.625% Guar 144A Sr Nts Due 2025 | 0.11 | 0.001068978 | 8 | 2728720.582 | 8 |
| CSC HOLDINGS LLC - 6.75% Sr Nts Ser B Due 2021 | 0.13 | 0.000778964 | 9 | 4496362.503 | 6 |
| CSC HOLDINGS LLC - 8.625% Sr Nts Ser B 2019 | 0.07 | 0.001284603 | 7 | 1860582 | 7 |
| UNITYMEDIA HESSEN & GMBH CO KG - 5% 144A Gtd Sr Sec Nts Due 2025 | 0.06 | 0.001742804 | 7 | 1905777.5 | 6 |
| UNITYMEDIA HESSEN & GMBH CO KG - 5.50% 144A Ser Sec Nts Due 2023 | 0.12 | 0.001798601 | 6 | 1651122.655 | 14 |
| VIDEOTRON LTD - 5.000% Ser B Sr Nts Due 2022 | 0.10 | 0.003371103 | 4 | 992619.2379 | 18 |
| VIDEOTRON LTD - 5.375% 144A Sr Nts Due 2024 | 0.07 | 0.002443855 | 5 | 1164075 | 11 |

Figure 6: Portfolio Optimization Constraints within Axioma Portfolio

Name: Tracking Error Minimization Date: 01/10/2017 Description: TE Import Information

Options | Objectives | Constraints | Hierarchy | Constraint Attribution

| Name | Active | Type | Selection | Scope | Unit | Min | Max | Benchmark |
|---|---|---|---|---|---|---|---|---|
| 100% Invested | ✓ | Limit Holding | MASTER | AGGREGATE | PERCENT | 100 | 100 | |
| Exposure 5% | ✓ | Limit Holding | MASTER | ASSET | PERCENT | 0 | 5 | |
| Liquidity 1-8 | ✓ | Limit Holding | Multiselection | ASSET | PERCENT | 0 | 0 | |
| Portfolio Duration | ✓ | Limit Holding | Effective Durati... | AGGREGATE | NUMBER | 0.25 | 0.5 | HYG Bmk |

Figure 7: Tracking Error Summary generated from Axioma Portfolio

| Summary Statistic | Initial Portfolio | Final Portfolio |
|---|---|---|
| **Risk** | | |
| Total Risk | 3.94% | 3.98% |
| Total Risk with Alpha Factor (15.0) | 3.95% | 4.20% |
| Total Factor Risk | 3.93% | 3.97% |
| Total Specific Risk | 0.35% | 0.32% |
| | | |
| Historical Beta (from risk model) | | |
| Predicted Beta (from model/benchmark) | 1.0000 | 1.0088 |
| Total Return at Risk (%) (5.0%) | 6.48% | 6.55% |
| Total Value at Risk ($) (5.0%) | $6,482,458.64 | $6,548,783.45 |
| Coefficient of Determination | 1.0000 | 0.9972 |
| | | |
| Benchmark Risk | 3.94% | 3.94% |
| Active Risk | 0.00% | 0.22% |
| Active Risk with Alpha Factor (15.0) | 0.00% | 1.44% |
| Active Factor Risk | 0.00% | 0.10% |
| Active Specific Risk | 0.00% | 0.19% |

that a certain sector or name will outperform other sectors or issuers, exposure can be increased on these bets within a risk budget. It is important to quantify these tilts or strategies using a risk budget and stress testing.

As an illustration we add additional constraints to Figure 6 where we overweight spread duration and OAS (option adjusted spread) to the Energy sector. Here we fix the exposure to the Energy sector, increase spread duration from 3.96 to 5.0, invest in energy bonds with an OAS greater than 500bps, and limit the total number of holdings to a maximum of 500. These additional constraints are listed in Figure 8. In Figure 9, the overweight in the Energy sector with the constraints listed above, resulted in an increase in standard deviation to 4.05% from 3.94% with an active risk of 31bps.

## Backtesting

Backtesting allows one to analyze and compare different optimization strategies (in addition to enabling one to access how well a risk model is performing) over time. We ran a replication strategy for an Emerging Market Sovereign portfolio denominated in USD on the first business day of every month between January 2008 and June 2015. The number of securities in the benchmark (our investable universe) in each rebalancing period is approximately 250. The objective is to minimize the tracking error with respect to the benchmark. We experiment with an unconstrained strategy (UC) as well as strategies that impose an upper bound on the number of instruments (100–200) that are held in the portfolio. For instance, the constrained strategy “Names100” ensures that the rebalanced portfolio holds no more than 100 names in any rebalancing period. Other constraints, including liquidity, exposure, and duration constraints, can also be added to the strategy in each period of the backtest. The portfolio is rolled forward using the realized portfolio returns between two successive rebalancings.

Figure 10 plots the time-series of the predicted (ex-ante) active risk for each of the strategies over the rebalancing period. Note that the UC strategy has a predicted tracking error close to zero while the most constrained strategy “Names100” has an average tracking error of 30bps over the rebalancing period. Figure 11 plots the rolling realized active risk for each of the strategies over the rebalancing period.

Notice that the rolling realized risk for the UC strategy is below 50bps on average, indicating that the risk model is doing an adequate job. On the other hand, the most constrained “Names100” strategy has an average annualized active risk of 70bps.

## Hedging and Immunization

Hedging a portfolio helps remove undesired exposures and mitigate anticipated market factors that would negatively impact a portfolio. Using stress tests in conjunction with risk models provides a more complete picture of risk. Stress tests are designed to estimate the impact of adverse market conditions on a portfolio, which risk models typically struggle to incorporate.

In Figure 12 we consider the impact of large spread shifts to our sample Emerging Market sovereign

Figure 8: Constraints for Portfolio Tilt

Name: Tracking Error Minimization Date: 01/10/2017 Description: TE Import Information

Options | Objectives | Constraints | Hierarchy | Constraint Attribution

| Name ↑ | Active | Type | Selection | Scope | Unit | Min | Max | Benchmark |
|---|---|---|---|---|---|---|---|---|
| 100% Invested | ✔ | Limit Holding | MASTER | AGGREGATE | PERCENT | 100 | 100 | |
| Energy Exposure Fixed | ✔ | Limit Holding | GICS Level 1_Ene... | SELECTION | PERCENT | 8.75 | 8.75 | |
| Energy OAS 5% | ✔ | Limit Holding | GICS Level 1_Ene... | AGGREGATE | NUMBER | 0.05 | 1 | |
| Energy Spread Duration 5-8 | ✔ | Limit Weighted Average Holding | Effective Dur - Gi... | SELECTION | NUMBER | 5 | 8 | |
| Exposure 5% | ✔ | Limit Holding | MASTER | ASSET | PERCENT | 0 | 5 | |
| Liquidity 1-8 | ✔ | Limit Holding | Multiselection | ASSET | PERCENT | 0 | 0 | |
| Max 500 Names | ✔ | Limit Names | MASTER | AGGREGATE | | | 500 | |
| Portfolio Duration | ✔ | Limit Holding | Effective Duration | AGGREGATE | NUMBER | 0.5 | 0.75 | HYG Bmk |

Figure 9: Tracking Error Summary for Tilt

| Summary Statistic | Initial Portfolio | Final Portfolio |
|---|---|---|
| **Risk** | | |
| Total Risk | 3.94% | 4.05% |
| Total Risk with Alpha Factor (15.0) | 3.95% | 4.38% |
| Total Factor Risk | 3.93% | 4.04% |
| Total Specific Risk | 0.35% | 0.33% |
| | | |
| Historical Beta (from risk model) | | |
| Predicted Beta (from model/benchmark) | 1.0000 | 1.0254 |
| Total Return at Risk (%) (5.0%) | 6.48% | 6.66% |
| Total Value at Risk ($) (5.0%) | $6,482,458.64 | $6,664,563.09 |
| Coefficient of Determination | 1.0000 | 0.9947 |
| | | |
| Benchmark Risk | 3.94% | 3.94% |
| Active Risk | 0.00% | 0.31% |
| Active Risk with Alpha Factor (15.0) | 0.00% | 1.75% |
| Active Factor Risk | 0.00% | 0.23% |
| Active Specific Risk | 0.00% | 0.21% |
| | | |
| Active Predicted Beta | 0.0000 | 0.0254 |
| Active Return at Risk (%) (5.0%) | 0.00% | 0.51% |
| Active Value at Risk ($) (5.0%) | $0.00 | $512,031.45 |
| Active Coefficient of Determination | 2.5368 | 0.1031 |
| Active Share | 0.0000 | 0.8653 |

Figure 10: Predicted active risk for backtest strategies

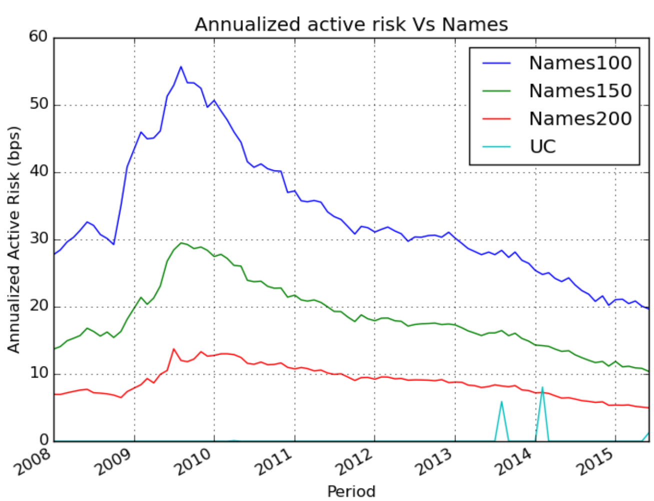
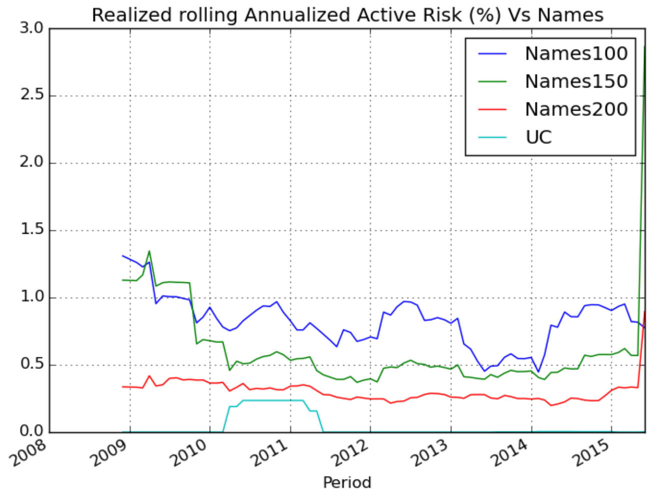


Figure 11: Realized Rolling Active Risk backtest strategies




portfolio. We display the impact of credit spread shifts of 100bps and 200bps, along with the CS01 sensitivity. In this example, the portfolio manager hedges against an anticipated deterioration of Brazilian sovereign bonds by purchasing CDS protection. In the report, we observe that the deterioration in bond prices is hedged by the CDS; a credit spread shift of 200bps results in a loss of 51bps (relative to the present value of the portfolio) for the sovereign bonds with an offsetting gain of 55bps from the CDS protection.

Another example is provided in Figure 13, where a US interest rate hike is anticipated. The last row represents a hedge with a short 10y bond future. The bond future position helps to mitigate a 100bps shift in interest rates by 65bps. In addition, VaR at the 95% confidence level is decreased from 8.26% to 8.01% (second to last column). The last column displays marginal VaR and highlights that the risk contribution from the bond future is negative, and thus a diversifying effect on the portfolio.

In both hedges above, we did not use an optimizer. In the first case, CDS protection was bought to match the CS01 sensitivity of bonds, and the second a budget of 10% from a market exposure perspective was implied. However, systematic or algorithmic procedures involving optimization techniques can be employed to select hedges. For instance, in [2], the downside risk of a portfolio is minimized with the addition of hedging overlays, specified by the user, which are constrained by a budget.

## Concluding Remarks

Although the construction and hedging of fixed-income portfolios is challenging, optimization techniques still help portfolio managers implement their investment strategies and overlay hedges. Index replication is difficult since most fixed-income indices contain a significant portion of illiquid securities whose fair market prices are difficult to estimate. In addition, bonds maturing, new bond issuances, and reinvestment of coupon payments all facilitate frequent rebalancing, which can be expensive and more tedious than equity index replication.

Even with these challenges, we can specify investment strategies by setting constraints to control (unwanted) exposures, liquidity, and transaction costs, implement factor and exposure tilts, and quantify deviation from a benchmark. Moreover, under a stress testing framework, an optimizer can be used to implement hedging and overlay strategies to mitigate tail risk. In this note, we provided examples involving portfolio construction and tilting portfolio views utilizing an optimizer.

Axioma continues to push forward on the fixed-income optimization front.

## References

[1] IDC, Liquidity Indicators Methodology, Product Note. https://www.theice.com/market-data/pricing-and-analytics/analytics/liquidity

[2] Sivaramakrishnan, K. and R. Stamicar (2016). A CVaR scenario-based framework for minimizing downside risk in multi-asset class portfolios. *Forthcoming, Journal of Portfolio Management, Winter 2018.*

Figure 12: Sovereign CDS Hedge

| Reporting Levels | CS01 | Credit Spreads +100bp | Credit Spreads +200bp |
|---|---|---|---|
| | USD | USD | USD |
| **EMB Mar 29 2017 +Overlay** | **-0.0613** | **-5.84** | **-10.61** |
| MX (3) | -0.0014 | -0.10 | -0.14 |
| RU (2) | -0.0029 | -0.27 | -0.52 |
| ID (1) | -0.0043 | -0.39 | -0.71 |
| TR (1) | -0.0035 | -0.32 | -0.60 |
| PH (1) | -0.0039 | -0.36 | -0.65 |
| AR (2) | -0.0025 | -0.24 | -0.39 |
| BR (3) | -0.0001 | 0.01 | 0.04 |
| Callable Bond (7) | -0.0020 | -0.19 | -0.35 |
| Bond (7) | -0.0009 | -0.09 | -0.16 |
| Credit Default Swap (1) | 0.0029 | 0.28 | 0.55 |
| CO (2) | -0.0037 | -0.34 | -0.62 |
| HU (1) | -0.0019 | -0.19 | -0.35 |
| ZA (2) | -0.0021 | -0.20 | -0.37 |
| PL (1) | -0.0016 | -0.15 | -0.30 |

Figure 13: Interest Rate Future Hedge

| Reporting Levels | Market Exposure (%) | OAS (bps) | IR +100bp | Duration | VaR95 | mVaR95 |
|---|---|---|---|---|---|---|
| | USD | USD | USD | USD | USD | USD |
| **EMB Mar 29 2017 +Overlay** | **81.28** | **324** | **-6.04** | **5.68** | **8.01** | **8.01** |
| EMB (352) | 91.27 | 288 | -6.70 | 6.99 | 8.26 | 8.38 |
| IR Hedge EMB (1) | -9.99 | 0 | 0.65 | -6.08 | 0.68 | -0.37 |

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